A divergence-free parametrization for dynamical dark energy

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Abstract

We introduce a new parametrization for the dark energy, led by the same idea to the linear expansion of the equation of state in scale factor a and in redshift z, which diverges neither in the past nor future and contains the same number of degrees of freedom with the former two. We present constraints of the cosmological parameters using the most updated baryon acoustic oscillation (BAO) measurements along with cosmic microwave background (CMB) data and a recent reanalysis of Type Ia supernova (SN) data. This new parametrization allowed us to carry out successive observational analyses by decreasing its degrees of freedom systematically until ending up with a dynamical dark energy model that has the same number of parameters with ΛCDM. We found that the dark energy source with a dynamical equation of state parameter equal $-2/3$ at the early universe and -1 today fits the data slightly better than Λ.

1 Introduction

The recent high precision data is in very good agreement with the six parameter base ΛCDM cosmology [1, 2]. However, we still cannot consider it as the final cosmological model. It may represent, rather, a very good approximation or a limiting case of a more general theory, which does not necessarily reduce to vacuum energy in terms of a cosmological constant Λ and may deviate from Λ considerably in the far future and/or past. Two well known problems related with the Λ assumption, the so called coincidence and fine-tuning problems, may be signs for a dynamical nature of dark energy [3, 4, 5, 6, 7, 8]. Some tensions between the Λ assumption and high precision data could be resolved in case of evolving dark energy (DE) (see for instance [9, 10, 11] and references therein).

Using the one parameter extension to the ΛCDM, wCDM adopts a spatially flat universe and constant equation of state (EoS) parameter w for DE, Planck collaboration [2] gives $w = -1.13^{+0.23}_{-0.25}$ from combined Planck+WP+highL+BAO data and BOSS collaboration [10] gives $w = -0.97 \pm 0.05$ from the most recent combined Planck+BAO+CMB data. Constraining the possible evolution of w on the other hand is difficult even with the powerful BAO, SN and CMB data that range at different redshift values. Hence, probing the possible DE evolution usually involves the introduction of a phenomenological parametrization for its EoS parameter involving a couple of free parameters. The most widely used parametrization for constraining evolving DE is the linear expansion in scale factor a which is known as CPL (Chevallier-Polarski-Linder [12, 13]) parametrization: $w(a) = w_0 + w_a(1 - a)$, where w_0 and w_a are real constants. It has been first proposed as an alternative to the linear redshift parametrization of DE: $w(z) = w_0 + w_z z$, which in contrast to CPL grows increasingly unsuitable at redshifts $z \gg 1$ and hence cannot be used with high-redshift data, e.g., CMB. However, even the tightest constraints on the parameters of CPL from combined Planck+BAO+SN allow a quite flexible range for the evolution of the EoS parameter for DE [10]. Besides this, compared to ΛCDM, the improvement in the success of fit to the data using this parametrization is not significant [2, 10, 11]. These might be signaling for that CPL is not an adequate choice for describing the possible evolution of dark energy. It is already mentioned in many studies that the divergent behavior of CPL not only prevents one to make plausible predictions on the future of the universe but also shows that it cannot genuinely cover theoretical models of dark energy. Throughout the literature there have been various DE parametrizations introduced by considering different reasoning and strategies for obtaining mathematically well behaving and physically acceptable models to obtain tighter constraints on DE and make plausible predictions on the future of the universe [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

In this paper we shall use a new parametrization for describing the DE source that may be taken as a natural extension of the same idea that give rise to linear redshift and CPL parametrizations that correspond to the first two terms of Taylor expansion in z and a, respectively. We start by introducing a fluid with an EoS parameter linear in time $\tilde{t}$. Then, we write its EoS parameter in terms of the scale factor $\tilde{a}$ by considering general relativity and utilize the

obtained $w(\tilde{a})$ for describing the DE component of the universe in our observational analyses. We carry out successive observational analyses by decreasing the number of degrees of freedom (DoF) until we end up with a dynamical DE model having no additional parameters compared to ΛCDM.

2 Dark energy parametrization

In this section we first derive an EoS parameter in terms of scale factor $\tilde{a}$, which yields linear EoS parameter in time $\tilde{t}$ when the universe is filled only with this fluid (here tilde denotes a universe contains only the fluid described by the EoS linear in time $\tilde{t}$). We then in Sec. 2.2 make use of this form of $w(\tilde{a})$ for describing the DE source in the physical universe that contains sources other than DE also, by adopting $\tilde{a} \rightarrow a$, where a represents the scale factor of the physical universe.

2.1 Equation of state parameter linear in time

Let us consider a fluid described by an EoS parameter expressed as a first order Taylor expansion in time $\tilde{t}$, that is:

$$w = w_0 + w_1(1 - \tilde{t}), \tag{2.1}$$

where w_0 and w_1 are real constants and $\tilde{t} > 0$ is the normalized time. It should be noted here that w would not diverge provided that the time is restricted as in the big rip* cosmologies. We construct a relation between the fluid described by the EoS parameter linear in time (2.1) and the scale factor by considering the general theory of relativity. Accordingly, we consider the Friedmann equations for spatially flat RW spacetime in the presence of a single fluid written as follows: $3\frac{\dot{a}^2}{\tilde{a}^2} = \kappa\rho$ and $\frac{\dot{a}^2}{\tilde{a}^2} + 2\frac{\ddot{a}}{\tilde{a}} = -\kappa p = -\kappa\rho w$, where a dot denotes derivative with respect to time $\tilde{t}$. Eliminating ρ between these two equations and then using (2.1) we get the corresponding deceleration parameter[†]

$$\tilde{q} \equiv -\frac{\ddot{a}\tilde{a}}{\dot{a}^2} = \frac{1}{2} + \frac{3}{2}\left[w_0 + w_1(1 - \tilde{t})\right], \tag{2.2}$$

whose solution, for $w_1 \neq 0$, reads

$$\tilde{a} = \tilde{a}_1 \exp\left[\frac{4}{3}\frac{\text{arctanh}\left(\frac{w_1\tilde{t}-(w_0+w_1+1)}{\sqrt{(w_0+w_1+1)^2-c_1 w_1}}\right)}{\sqrt{(w_0 + w_1 + 1)^2 - c_1 w_1}}\right], \tag{2.3}$$

where $\tilde{a}_1$ and c_1 are integration constants. For the case $w_1 = 0$, we would get the standard power law solution as $\tilde{a} \propto \tilde{t}^{2/3(1+w_0)}$ with $\rho \propto \tilde{a}^{-3(1+w_0)}$. We demand a big bang $\tilde{a} = 0$ at $\tilde{t} = 0$, and expanding universe $\dot{a} \geq 0$ for $\tilde{t} \geq 0$ and $\tilde{a} \rightarrow +\infty$ at a finite time, say, as $\tilde{t} \rightarrow \tilde{t}_{\text{BR}}$ where $\tilde{t}_{\text{BR}}$ is the big rip time, so the EoS parameter never diverges. One may check that the first condition is satisfied by choosing $c_1 = 0$, and in addition to this the latter two conditions imply the following relations $2 + 2w_0 + 2w_1 > w_1 > 0$. Accordingly, substituting $c_1 = 0$ in (2.3), our solution reduces to

$$\tilde{a} = \tilde{a}_1 \exp\left[\frac{4}{3}\frac{\text{arctanh}\left(\frac{w_1\tilde{t}}{w_0+w_1+1} - 1\right)}{w_0 + w_1 + 1}\right], \tag{2.4}$$

with $2 + 2w_0 + 2w_1 > w_1 > 0$. Isolating $\tilde{t}$ in this solution by setting $\tilde{a}(\tilde{t} = 1) = 1$ we obtain

$$\tilde{t}(\tilde{a}) = \frac{2(1 + w_0 + w_1)}{w_1 + (2 + 2w_0 + w_1)\tilde{a}^{\frac{-3(1+w_0+w_1)}{2}}}. \tag{2.5}$$

Finally using $\tilde{t}(\tilde{a})$ in (2.1) we obtain the following EoS in scale factor $\tilde{a}$

$$w(\tilde{a}) = w_0 + w_1\left[1 - \frac{2(1 + w_0 + w_1)}{w_1 + (2 + 2w_0 + w_1)\tilde{a}^{\frac{-3(1+w_0+w_1)}{2}}}\right] \tag{2.6}$$

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*See [28] for big rip cosmology.

[†]It may be noteworthy that a deceleration parameter linear in time for the observed universe was obtained from a higher dimensional cosmological model in dilaton gravity in [29], where its value at $\tilde{t} = 0$ is depending on the number of extra dimensions.

with $2 + 2w_0 + 2w_1 > w_1 > 0$. Next, using the energy-momentum conservation equation we obtain the energy density ρ:

$$\rho = \rho_0 \tilde{a}^{-3(w_0+w_1+1)} \left(1 + w_1 \frac{\tilde{a}^{\frac{3}{2}(w_0+w_1+1)} - 1}{2w_0 + 2w_1 + 2} \right)^4, \tag{2.7}$$

where $\rho_0 = \rho(\tilde{a} = 1)$. We notice that $w \to \frac{-3(w_0+1)\ln(\tilde{a}) - 4w_0}{3(w_0+1)\ln(\tilde{a}) - 4}$ and $\rho \to \rho_0 \left[-1 + \frac{3(w_0+1)}{4} \ln(\tilde{a}) \right]^4$ as $w_1 \to -1 - w_0$.

The EoS parameter (2.6) is bounded both as $\tilde{a} \to 0$ and $\tilde{a} \to \infty$, namely, $w \to w_0 + w_1$ as $\tilde{a} \to 0$ and $w \to -w_0 - w_1 - 1$ as $\tilde{a} \to \infty$. We note that $\tilde{a} \to 0$ as $\tilde{t} \to 0$ and $\tilde{a} \to \infty$ as $\tilde{t} \to 2(1 + w_0 + w_1)/w_1$, hence the time could not take arbitrarily large values unless $w_1 = 0$, which also explains why EoS parameter linear in $\tilde{t}$ does not diverge in future in this case.

The recent high precision data is in very good agreement with Λ as the DE source in the universe. However, it may represent, rather, a very good approximation or a limiting case of a more general theory, which does not reduce necessarily to vacuum energy in terms of a cosmological constant. In this regard, being $w_{\text{de}} \sim -1$ is in very good agreement with precision data, it seems physically more acceptable to consider an approximate description of possibly dynamical DE with a parametrization that is mathematically well behaved and may never depart from $w_{\text{de}} \sim -1$ a lot. We note that the EoS parameter we derived has these properties and hence may be adopted for parametrizing the EoS of the DE source in the universe containing other sources also.

2.2 Parametrization of dark energy source

The EoS derived above was first given in [30] starting from the linear deceleration parameter in time using general theory of relativity and then utilized in [31, 32] for describing the effective EoS parameter averaging all the ingredients of the universe to constrain particularly the kinematical properties of the expansion of the universe. In this work, on the other hand, we follow a quite different method and adopt the EoS written in terms of $\tilde{a}$ for describing only the DE component of the universe, rather than the average ingredient of the universe. The cost of this is that (2.6) will not correspond to the linear EoS in physical time t in the actual universe anymore, unless the universe is dominated by this fluid. The reason being is that including the other sources such as cold dark matter (CDM) in addition to DE the evolution of the scale factor will deviate from the one given in (2.4), i.e., the relation between the scale factor a and time t will be different, until the DE source becomes dominant over all other sources in the universe.

Let us now adopt the EoS parameter given in (2.6) for describing DE source by setting $\tilde{a} \to a$ and then write it in a more useful form by defining $w_{\text{i}} = w_0 + w_1$:

$$w_{\text{de}} = w_{\text{i}} - \frac{2(w_{\text{i}} - w_0)(1 + w_{\text{i}})}{(w_{\text{i}} - w_0) + (2 + w_{\text{i}} + w_0) a^{-\frac{3}{2}(1+w_{\text{i}})}}, \tag{2.8}$$

which also recasts the energy density (2.7) as

$$\rho_{\text{de}} = \rho_{\text{de}}^{(0)} \left[\frac{w_{\text{i}} - w_0}{2w_{\text{i}} + 2} a^{\frac{3}{4}(w_{\text{i}}+1)} + \frac{w_{\text{i}} + w_0 + 2}{2w_{\text{i}} + 2} a^{-\frac{3}{4}(w_{\text{i}}+1)} \right]^4, \tag{2.9}$$

where a is the physical scale factor, and the conditions on the parameters are now given by $-2 - w_{\text{i}} < w_0 < w_{\text{i}}$. Our parametrization has the same number of degrees of freedom with the CPL but while the variation of the EoS of DE with respect the scale factor is constant in scale factor $w'_{\text{CPL}} = -w_a$ in CPL, here it is a dynamical quantity:

$$w'_{\text{de}} = \frac{3(w_{\text{i}} + 1)^2 [(w_0 + 1)^2 - (w_{\text{i}} + 1)^2] a^{\frac{3}{2}w_{\text{i}} + \frac{1}{2}}}{\left[(w_0 - w_{\text{i}}) a^{\frac{3}{2}w_{\text{i}} + \frac{3}{2}} - (w_0 + w_{\text{i}} + 2) \right]^2}, \tag{2.10}$$

where a prime denotes derivative with respect to the scale factor a.

We note that the form of the energy density (2.9) we obtained by the reasoning we followed in the previous section has a symmetry for $w_{\text{i}} + 1 \to -w_{\text{i}} - 1$ and hence it is enough if we consider the cases $w_{\text{i}} > -1$ only. In accordance with this, we see from (2.8) that $w_{\text{de}} \to w_{\text{i}}$ as $a \to 0$ $(z \to \infty)$ and hence w_{i} denotes the initial value of the EoS parameter of DE. Similarly $w_{\text{de}} \to -w_{\text{i}} - 2$ as $a \to \infty$ $(z \to -1)$, and hence we can write the final value of the EoS parameter as $w_{\text{f}} = -w_{\text{i}} - 2$. We also note that w'_{de} can take only negative values as long as we stick to the range $-w_{\text{i}} - 2 < w_0 < w_{\text{i}}$ for the parameters, but take positive values if this condition could be violated. The radiation and pressure-less matter components of the universe enforce universe to expand as $a \propto t^{\frac{1}{2}}$ and $a \propto t^{\frac{2}{3}}$, respectively. The DE source described by (2.9), on the other hand, enforces universe to expand according to the kinematics we discussed in the previous

section. In the physical universe, say, in the presence of all these components, the expansion of the universe will be much complicated as it will be determined by the joint effect of these components. We also note that the conditions on the parameters of the hypothetical fluid, we are now utilizing for describing DE, were initially considered to obtain a physically reasonable expansion history for the universe filled by this fluid only. However the presence of radiation and pressure-less matter in addition to a DE source described by (2.9) can lead to viable expansion histories for the universe even if the parameters w_i and w_0 take values out of the range $-w_i - 2 < w_0 < w_i$ and hence provides us freedom to violate this condition. Accordingly, in the next section we will carry out observational analyses without taking this condition into account, so that our analyses will not be restricted to the cases $w'_{de} < 0$. However we will see below that the concluding values for w_i and w_0 upon the three successive observational analyses, where we decrease the number of degrees of freedom systematically, satisfy the condition $-w_i - 2 < w_0 < w_i$.

3   Observational constraints

We consider a spatially flat RW spacetime and the Hubble parameter H in the presence of radiation Ω_r, pressure-less fluid/(CDM+matter) Ω_m and the dark energy Ω_{de} described by (2.9) is given by

$$\frac{H^2}{H_0^2} = \Omega_r^{(0)}(1+z)^4 + \Omega_m^{(0)}(1+z)^3 + \Omega_{de}^{(0)}\left[\frac{w_i - w_0}{2w_i + 2}(1+z)^{-\frac{3}{4}(w_i+1)} + \frac{w_i + w_0 + 2}{2w_i + 2}(1+z)^{\frac{3}{4}(w_i+1)}\right]^4, \quad (3.1)$$

where the density parameter of the DE $\Omega_{de}^{(0)} = 1 - \Omega_m^{(0)} - \Omega_r^{(0)}$ and $w_i > -1$. Here subscript/superscript 0 indicates the values of the parameters today. The density parameter of radiation is $\Omega_r^{(0)} = 2.469 \times 10^{-5}h^{-2}(1 + 0.2271N_{eff})$, where $h = H_0/100\text{kms}^{-1}\text{Mpc}^{-1}$ [1]. We consider a model with standard matter and radiation content, including three neutrino species with minimum allowed mass $\sum m_\nu = 0.06\,\text{eV}$. Throughout the analysis we assume flat priors over our sampling parameters: $w_0 = [-2.0, -0.5]$, $w_i = [-1.0, 2.0]$ for the dark energy EoS parameters, $\Omega_m^{(0)} = [0.05, 1]$ for the pressure-less matter density parameter today, $\Omega_b^{(0)}h^2 = [0.02, 0.025]$ for the baryon density today and $h = [0.4, 1.0]$ for the reduced Hubble constant. It might be noteworthy to note that the radiation energy density $\rho_r^{(0)} = \Omega_r^{(0)}H_0^2$ today is not subject to our analysis since it is well constrained, such that it has a simple $\rho_r = \frac{\pi^2}{15}T_{CMB}^4$ relation with the CMB monopole temperature (see [33] for further details), which today is very precisely measured to be $T_{CMB}^{(0)} = 2.7255 \pm 0.0006\,\text{K}$ [34].

In order to perform the parameter space exploration, we make use of a modified version of a simple and fast Markov Chain Monte Carlo (MCMC) code that computes expansion rates and distances from the Friedmann equation, named SimpleMC. This code already contains a compressed version of the Planck data, a recent reanalysis of Type Ia supernova (SN) data, and high-precision BAO measurements at different redshifts up to $z < 2.36$ [10].

In three successive subsections we carry out observational analyses: i) w_0 and w_i are both free parameters: DoF are the same with CPL, ii) w_0 is free but w_i is fixed to certain values, so that DoF are the same with wCDM, and iii) both w_0 and w_i are fixed to certain values so that DoF are the same with ΛCDM.

3.1   General case: free w_0 and w_i

We first constrain the cosmological parameters by taking both w_0 and w_i as free parameters so that we have the same number of DoF with the CPL parametrization. We summarize the results for the data sets Planck+BAO, Planck+SN and Planck+BAO+SN in Table 1. In Figure 1 we show 1-D and 2-D posterior distributions of w_0 and w_i for the following data sets Planck+BAO, Planck+SN and Planck+BAO+SN. In Figure 2 we give 2-D posterior distributions for $w_0 - h$ and $w_0 - \Omega_m^{(0)}$.

We observe that the matter density parameter $\Omega_m^{(0)}$ and the reduced Hubble constant h are obtained almost the same in three combinations of data sets and Planck+BAO+SN data set leads to the tightest constraints; $\Omega_m^{(0)} = 0.304 \pm 0.009$ and $h = 0.677 \pm 0.011$.

In all cases the best-fit parameters are consistent with ΛCDM model, i.e., $(w_0, w_i) = (-1, -1)$. We observe that Planck+SN data lead to a tighter constraint on w_0 while Planck+BAO data lead a tighter constraint on w_i. The central value of w_0 is almost equal to -1 from both Planck+BAO, Planck+SN and Planck+BAO+SN and its value from the different data combinations changes by less than 1σ. Using three data sets at the same time we obtain $w_0 = -0.99 \pm 0.06$ (Planck+BAO+SN) with 1σ having the tightest constraint on w_0 with a central value almost exactly equal to -1. Using CPL, on the other hand, w_0 is not constrained very well and the central value of w_0 is obtained not only quite higher than -1 but also significantly different from three different data combinations;

Table 1: Parameter constraints. SN data cannot constraint w_i as good as w_0 while it is the other way around for BAO data. For two-tailed distributions the results are given in 1σ and for one-tailed distributions given in 2σ.

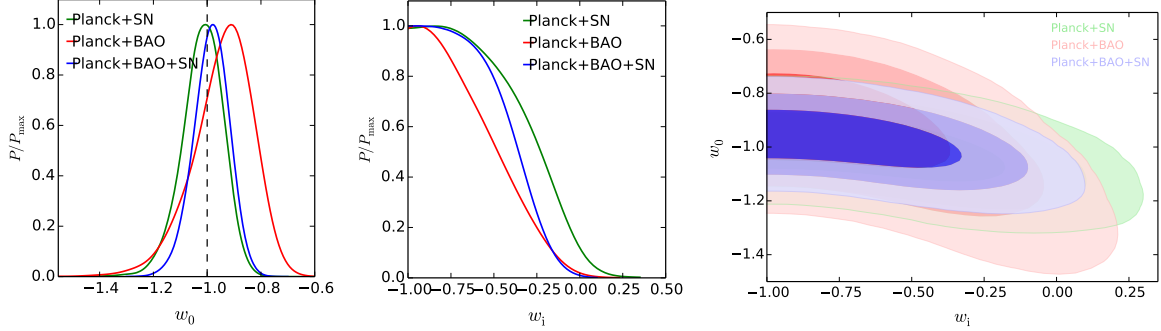| Data set	$\Omega_\mathrm{m}^{(0)}$	h	w_0	$-1 < w_\mathrm{i}$
Planck+BAO	$0.306^{+0.014}_{-0.015}$	$0.673^{+0.020}_{-0.018}$	-0.97 ± 0.10	< -0.30
Planck+SN	$0.304^{+0.021}_{-0.019}$	0.677 ± 0.019	$-1.02^{+0.07}_{-0.08}$	< -0.27
Planck+BAO+SN	0.304 ± 0.009	0.677 ± 0.011	-0.99 ± 0.06	< -0.42

Figure 1: 1-D and 2-D posterior distributions for the parameters w_0 and w_i; data combinations are indicated in the legends.
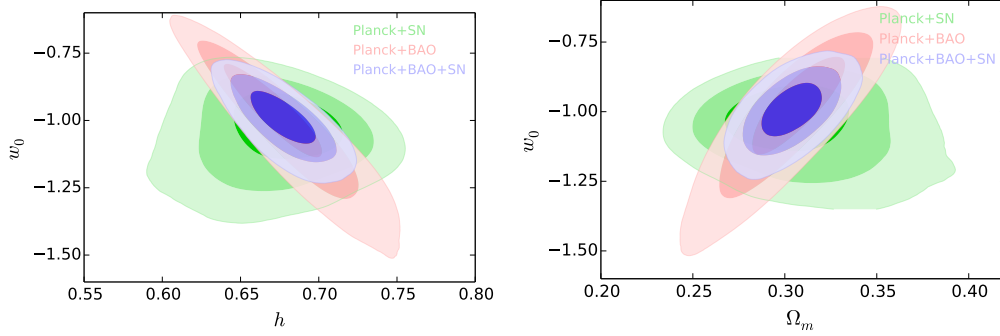

Figure 2: 2-D posterior distributions for $w_0 - h$ and $w_0 - \Omega_\mathrm{m}^{(0)}$; data combinations are indicated in the legends.

$w_0 = -0.58 \pm 0.24$ (Planck+BAO), $w_0 = -0.90 \pm 0.16$ (Planck+SN) and $w_0 = -0.93 \pm 0.11$ (Planck+BAO+SN) [10]. Constraint on w_i from Planck+SN is quite loose such that $-1 < w_\mathrm{i} < -0.27$ (2σ); so that the DE could either behave like cosmic strings or a cosmological constant at very high red-shift values. Planck+BAO data constrain w_i better and constrict the allowed range to $-1 < w_\mathrm{i} < -0.30$ (2σ). The tightest constraint is obtained from Planck+BAO+SN data; $-1 < w_\mathrm{i} < -0.42$ (2σ). According to this initially dark matter and cosmic strings like DE scenarios are not viable. Considering the constraints on w_0 and w_i together we observe that Λ as the DE source is doing great but even considerably large deviations from Λ are still allowed, for instance, $(w_0, w_\mathrm{i}) = (-1, -\frac{2}{3})$ is also perfectly allowed in this picture. The best fitting model to Planck+BAO+SN data has $\chi^2 = 46.63$, representing an improvement of $\Delta\chi^2 = 0.37$ compared to ΛCDM for which $\chi^2_{\Lambda\mathrm{CDM}} = 47.00$, for the addition of two extra parameters. This is an improvement almost the same with the one in the CPL model $\Delta\chi^2_{\mathrm{CPL}} = 0.42$ [10]. Accordingly in terms of information criteria the improvement is not sufficient enough to justify the addition of two extra degrees of freedom either in our parametrization or CPL parametrization and additionally there is no reason to prefer one over the other among these two parametrizations.

3.2 Free w_0 and fixed w_i

We may try to get more information using our parametrization by reducing its DoF to the that of the wCDM parametrization by fixing w_i. We now carry out the observational analyses using Planck+BAO+SN data by setting either $w_i = 0$ (DE starts like pressure-less matter), $w_i = -\frac{1}{3}$ (DE starts like cosmic strings) or $w_i = -\frac{2}{3}$ (DE starts like cosmic domain walls). In Table 2, we summarize the results, including the minimum χ^2 values, from Planck+BAO+SN data. For a comparison, the constraint on wCDM parametrization from Planck+BAO+SN data is $w_0 = -0.97 \pm 0.08$ (1σ) [10]. We give the 1-D probability distributions of w_0 in our model in Figure 3. We note that the central value of w_0 shifts from the values less than -1 to the values higher than -1 and the χ^2 improves as w_i goes from 0 to -1. Given that the case $w_i = 0$ yields very large χ^2 relative to other three, it can be rejected. The most interesting point in Table 2 is that w_0 takes the closest value to -1 in the case $w_i = -\frac{2}{3}$ rather than in the case $w_i = -1$ and these two cases have almost the same χ^2 values that are considerably lower than the cases $w_i = 0$ and $w_i = -\frac{1}{3}$. This may be interpreted as a signal for a DE with a dynamical EoS parameter starting from values $\sim -\frac{2}{3}$ at earlier times and then approaching a cosmological constant as the universe expands, as noted also by [35].

Table 2: Mean values with 1σ errors of the parameters of the model for the particular values $w_i = 0$, $w_i = -\frac{1}{3}$, $w_i = -\frac{2}{3}$ and $w_i = -1$ from Planck+BAO+SN data.

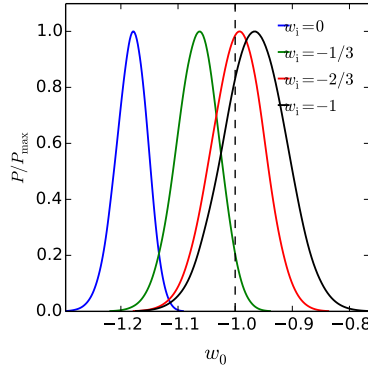| $\Omega_{\mathrm{m}}^{(0)}$	h	w_0	w_i	χ^2
0.289 ± 0.008	0.688 ± 0.010	$-1.180^{+0.028}_{-0.025}$	0	54.04
0.298 ± 0.008	0.680 ± 0.010	$-1.065^{+0.037}_{-0.035}$	$-\frac{1}{3}$	48.44
0.303 ± 0.009	$0.677^{+0.009}_{-0.011}$	$-0.995^{+0.046}_{-0.042}$	$-\frac{2}{3}$	46.90
0.305 ± 0.009	$0.676^{+0.010}_{-0.011}$	-0.966 ± 0.053	-1	46.64

Figure 3: 1-D posterior distributions for w_0 for the particular values $w_i = 0$ (blue), $w_i = -\frac{1}{3}$ (green), $w_i = -\frac{2}{3}$ (red) and $w_i = -1$ (black) from Planck+BAO+SN data.

3.3 Fixed w_0 and fixed w_i

We observe from Table 2 that w_0 persists on yielding values around -1 in spite of the large differences between the fixed values of w_i. Hence we now fix also $w_0 = -1$ along with $w_i = 0$, $w_i = -\frac{1}{3}$, $w_i = -\frac{2}{3}$ or $w_i = -1$. Doing so our model now yields the same number of DoF with the ΛCDM model and the latter case $(w_0, w_i) = (-1, -1)$ corresponds to Λ while the former three cases correspond to dynamical DE models. We summarize the constraints from Planck+BAO+SN data set with their minimum χ^2 values in Table 3. We note that the case $(w_0, w_i) = (-1, 0)$ and $(w_0, w_i) = (-1, -\frac{1}{3})$ should be ruled out due to their very large χ^2 values. We note on the other hand that the cases $(w_0, w_i) = (-1, -\frac{2}{3})$ and $(w_0, w_i) = (-1, -1)$, i.e. ΛCDM, have almost the same low χ^2 values; $\chi^2_{(-1,-2/3)} = 46.88$ and $\chi^2_{\Lambda\mathrm{CDM}} = 47.00$, respectively. However, it is noteworthy to note the case $(w_0, w_i) = (-1, -\frac{2}{3})$ has the lowest χ^2

Table 3: Mean values with 1σ errors of the parameters of the model with $w_0 = -1$ for the particular values $w_\mathrm{i} = 0$, $w_\mathrm{i} = -\frac{1}{3}$ and $w_\mathrm{i} = -\frac{2}{3}$ and χ^2 values from Planck+BAO+SN data. The case $w_\mathrm{i} = -1$ corresponds to ΛCDM.

$\Omega_\mathrm{m}^{(0)}$	h	w_0	w_i	χ^2
$0.316^{+0.006}_{-0.005}$	0.605 ± 0.002	-1	0	173.34
$0.305^{+0.007}_{-0.008}$	0.664 ± 0.005	-1	$-\frac{1}{3}$	51.80
$0.302^{+0.007}_{-0.008}$	0.678 ± 0.006	-1	$-\frac{2}{3}$	46.88
0.302 ± 0.008	0.682 ± 0.006	-1	-1	47.00

value with a difference over the ΛCDM given by $\chi^2_{\Lambda\mathrm{CDM}} - \chi^2_{(-1,-2/3)} = 0.12$, although not statistically significant. In this case we find from (2.10) that $w'_\mathrm{de} < 0$ through the history of universe and $w'_\mathrm{de}(z = 0) = -\frac{1}{12}$ today, while w' is always null in the case of Λ. According to this we have a dynamical DE model that fits to data equally better than Λ but yet does never cover/mimic Λ. We plot w_de and $\mathrm{d}w_\mathrm{de}/\mathrm{d}z = -w'_\mathrm{de}/(1 + z)^2$ versus redshift z in Fig. 4 for demonstrating how the dynamics of DE in case $(w_0, w_\mathrm{i}) = (-1, -\frac{2}{3})$ deviate from Λ.

Two key parameters in cosmology are the deceleration and jerk parameters that are defined as $q = -\frac{\ddot{a}}{aH^2} = \frac{(1+z)\mathrm{d}H}{H\mathrm{d}z} - 1$ and $j = \frac{\dddot{a}}{aH^3} = q + 2q^2 + (1+z)\frac{\mathrm{d}q}{\mathrm{d}z}$, respectively. The negative values of deceleration parameter imply that the universe is expanding with an accelerating rate and values less than -1 indicate a super-exponential expansion. The jerk parameter on the other hand is a very useful parameter for investigating the deviation of a cosmological model from ΛCDM since its value stays pegged to unity in ΛCDM (ignoring the contributions other than Λ and pressure-less matter), while it is in general a dynamical quantity in our model. In the case $(w_0, w_\mathrm{i}) = (-1, -\frac{2}{3})$ the deceleration parameter evolves from 0.5 as in the ΛCDM and goes ever monotonically to $-\frac{3}{2}$ which indicates a big rip end of the universe while the universe approaches asymptotically de Sitter universe with a deceleration parameter equal to -1 in ΛCDM model. We depict the evolution of q and j in redshift z for $(w_0, w_\mathrm{i}) = (-1, -\frac{2}{3})$ and ΛCDM in Fig 5 using the values with 1σ errors from Table 3. We find that the current value of the deceleration parameter is $q_0 = -0.545 \pm 0.014$ (1σ) and the universe starts its accelerating expansion at redshift value $z_\mathrm{tr} = 0.647^{+0.021}_{-0.018}$ (1σ) while these values are $q_0 = -0.547 \pm 0.012$ (1σ) and $z_\mathrm{tr} = 0.665 \pm 0.021$ (1σ) in ΛCDM model. We observe that the jerk parameter exhibits a non-monotonic behavior but stays at values very close to unity, and find its current value is $j(z = 0) = 1.087 \pm 0.001$ (1σ). Although the expansion history of the universe is almost the same in both models, they predict completely different futures. In the case $(w_0, w_\mathrm{i}) = (-1, -\frac{2}{3})$ the universe enters into a super-acceleration regime $(q < -1)$ at redshift $z_\mathrm{s} = -0.487^{-0.003}_{+0.004}$ (1σ) and ends with a big rip.
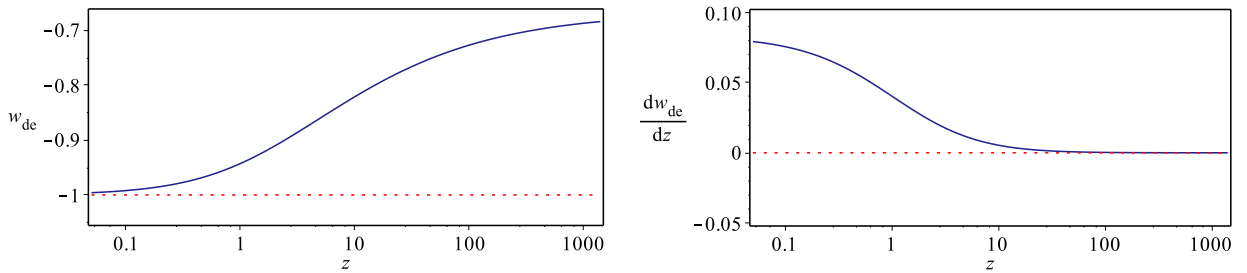

Figure 4: w_de and $\mathrm{d}w_\mathrm{de}/\mathrm{d}z$ in terms of redshift z for the case $(w_0, w_\mathrm{i}) = (-1, -\frac{2}{3})$ (solid lines). The dotted lines correspond to the case cosmological constant.

4 Concluding remarks

The new parametrization of DE we introduced in this paper allowed us to carry out successive observational analyses by decreasing its degrees of freedom systematically until ending up with a dynamical dark energy model having no additional parameters compared to ΛCDM. We first fixed the initial value of the EoS to some particular values and
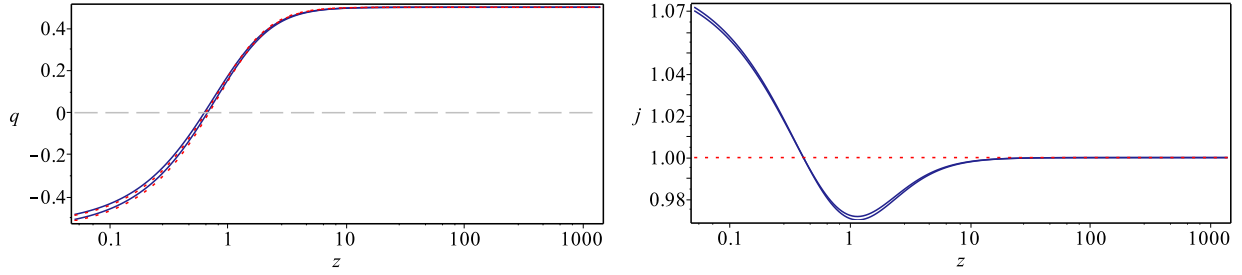
Figure 5: Deceleration parameter q and jerk parameter j versus redshift z for the case $(w_0, w_i) = (-1, -\frac{2}{3})$ (solid lines) and ΛCDM (dotted lines).

found some indications in favor of an evolving DE. We then additionally fixed the EoS parameter to -1 for the present epoch of the universe and found that EoS parameter that equals $-\frac{2}{3}$ at high redshifts and -1 at the present epoch gives a slightly better fit to the combined Planck+BAO+SN data than Λ. The variation of the EoS parameter in scale factor, on the other hand, is dynamical but always less than zero, which means that even today, when its EoS parameter is equal to -1, the DE does not genuinely mimic Λ. Thus we gave a cosmological model that can compete with ΛCDM but involving a dynamical DE. We also note that DE in this model passes below the phantom divide line in the future, hence leading to a big rip end of the universe. Accordingly, because the universe lives for finite time and the EoS parameter of DE evolves around -1, hence the energy density of DE is also dynamical, throughout the history of universe, our model relieves also the problems related with cosmological constant assumption of ΛCDM model.

Acknowledgments

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